



# PhD proposal: A Diagrammatic Quantum Simulator for Spin Liquids

Pascal Viot, Laura Messio, Riccardo Rossi  
(LPTMC, Sorbonne Université)

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## Context: the challenge of simulating entangled states of frustrated quantum spins

The quantum spin-1/2 Heisenberg hamiltonian:

$$H = \sum_{\langle i,j \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

is a paradigmatic model for crystalline compounds and cold-atom systems, hosting fascinating quantum phases of matter such as long-range ordered phases and topological spin liquids [1]. Despite its simplicity, reliable solutions are not yet available in the most interesting cases involving frustration of the spin interactions. Traditional numerical methods like Quantum Monte Carlo are not efficient for frustrated systems [3], while Tensor Networks approaches like DMRG [6] face challenges representing two and three dimensional quantum entanglement. Mean-field theories based on bosonic and fermionic [4, 5] parton operators are a standard tool to describe spin-liquid phases. However, it is hard to compute corrections to these traditional mean-field theories as additional constraints need to be imposed.

## Scientific goals

In this project, we propose to investigate an unconstrained fermionic representations for spin particles and explore its use to describe spin-liquid phases at both qualitative and quantitative level. In the first part of the thesis the student will inspect mean-field theories in this representation, with particular attention to gauge equivalence between different mean-field ansatze, and analytically treat simple cases. Feynman-diagram rules for the expansion around mean-field theory will be derived, and the first corrections will be estimated. In the second part of the project, a generic Diagrammatic Monte Carlo code for quantum spins based on fermionization will be implemented, which will be used to investigate frustrated quantum spin models that are not reachable with other techniques. This will be based on new developments in Diagrammatic Monte Carlo techniques [2]

The main scientific goal is to provide new controlled numerical tools to simulate frustrated quantum spin models, and then use these tools to study systems relevant to condensed-matter physics experiments. An additional goal is to provide new ways to benchmark quantum simulation platforms like Rydberg atoms or digital quantum computers.

# Justification of the scientific approach and place in the QICS initiative

Building a systematic diagrammatic expansion around a fermionic spin-liquid mean-field theory is a promising approach at least when the ground state is a spin liquid. The fact of being able to simulate finite temperature properties allows us to have reliable benchmarks from the high-temperature series expansion. The approach will be extensively benchmarked with exact diagonalization and Quantum Monte Carlo techniques, where available.

One of the main areas of application of quantum information technologies is the simulation of quantum systems. It is believed that providing new numerical tools to benchmark these technologies could be important for further developments [7].

## Role of supervisors

The experience of Messio in the field of frustrated magnetism will be needed to guide the student on the physical part and on the applications. Rossi and Viot will help in the development of the diagrammatic formalism and in the numerical implementation.

## Required skills

The ideal candidate has a strong background in theoretical physics, particularly quantum field theory. Experience with numerical tools is also valuable.

## References

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